Long-range dependence in returns and volatility of Central European Stock Indices*

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Abstract. In the paper, we research on the presence of long-range dependence in returns and volatility of Hungarian (BUX), Czech (PX) and Polish (WIG) stock indices between years 1997 and 2009 with a use of classical and modified rescaled range and rescaled variance analyses. Moving block bootstrap with pre-whitening and post-blackening is used for a construction of confidence intervals for the hypothesis testing. We show that there is no significant long-range dependence in returns of all examined indices. However, significant long-range dependence is detected in volatility of all three indices. The results for returns are contradictory with several studies which claim that developing markets are persistent. However, majority of these studies either do not use the confidence intervals at all or only the ones based on standard normal distribution. Therefore, the results of such studies should be reexamined and reinterpreted.

Keywords: Long-range dependence, bootstrapping, rescaled range analysis, rescaled variance analysis

JEL classification: C4, C5, G15

1. Introduction

Long-range dependence was examined and claimed to be found in several different assets such as stock indices [11, 12, 30, 29], interest rates [10, 5], government bonds [6, 12], exchange rates [35], electricity prices [2] and commodities [31] using various methods. Majority of research papers interpret the presence of long-range dependence on a basis of comparison of the long-range dependence characteristic parameter - Hurst exponent H - with the critical value of 0.5. Without further discussion and analysis, such approach is questionable as a significant part of the most popular Hurst exponent H estimation techniques are biased by a presence of short-term memory, e.g. $ARIMA$ or $GARCH$, in the underlying process. However, distinguishing between short and long-range dependence is essential for the financial

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analysis as both types have different implications for important financial areas such as a portfolio selection, an option pricing, a risk management and an asset pricing.

For the portfolio selection, the presence of long-range dependence implies infinite variance and thus a failure of the traditional portfolio models. Additionally, the traditional models assume short-range correlations only. Standard Black-Scholes model for option pricing has to be adjusted as well as the long-range dependence in either returns or volatility influences the measures of volatility in the model. As the variance is infinite or not defined for long-range dependent series, the classical risk management tools fail as well. Moreover, an existence of the long-range dependence in returns of assets implies better predictability and thus possibility of arbitrage. Finally, if either returns or volatility are long-range dependent, autoregressive fractionally integrated moving average models ($ARFIMA$) or fractionally integrated generalized autoregressive conditional heteroskedasticity models ($FIGARCH$) or combined are more precise than $ARIMA$ and $GARCH$ models.

To efficiently distinguish between the short and the long-range dependence, we use classical and modified rescaled range analyses [18, 26] and rescaled variance analysis [16]. For a construction of confidence intervals for a testing of no long-range dependence hypothesis, moving block bootstrap with pre-whitening and post-blackening procedure is used [32].

In the paper, we focus on detection of long-range dependence in returns and volatility of the stock indices of the Czech Republic (PX), Hungary (BUX) and Poland (WIG). The data set covers the evolution for the periods of 26.7.2001 - 30.6.2009 for BUX, 7.7.1997 - 30.6.2009 for PX and 14.10.2003 - 30.6.2009 for WIG.

The paper is structured as follows. In Section 2 and 3, the long-range dependence is shortly introduced together with rescaled range, modified rescaled range and rescaled variance analyses. Section 4 describes research methodology with a focus on the moving block bootstrap and a choice of its parameters. In Section 5, the data set is briefly examined. Section 6 presents the results and asserts that there is no long-range dependence in returns of any examined index whereas there is strong long-range dependence in volatility of all examined indices. Section 7 concludes.

2. Long-range dependence

The long-range dependence is present in stationary time series if autocorrelation function ρ of said process decays as $\rho(k) \approx Ck^{2H-2}$ for lag k approaching infinity. Parameter $0 < H < 1$ is called Hurst exponent after water engineer Harold Edwin Hurst who used the exponent for description of river flows behavior of the Nile River [18, 27].

A critical value of Hurst exponent is 0.5 and suggests two possible processes. H being equal to 0.5 implies either an independent process [3] or a short-term dependent process [25]. Independent process has zero auto-covariances at all non-zero lags. On the other hand, short-term dependent process shows non-zero auto-covariances at low lags which decay exponentially to zero at high lags.

If $H > 0.5$, auto-covariances of the process are significantly positive at all lags so that the process is called long-range dependent with positive correlations [15] or persistent [27]. Auto-covariances of such process are hyperbolically decaying and non-summable so that $\sum_{k=0}^{\infty}\gamma(k) = \infty$ [3]. On the other hand, if $H < 0.5$, auto-covariances are significantly negative at all lags and the process is said to be long-range dependent with negative correlations [15] or anti-persistent [27]. Similarly to the previous case, auto-covariances of such process are hyperbolically decaying but summable so that $0 < |\sum_{k=0}^{\infty}\gamma(k)| < \infty$ [15]. The persistent process implies that a positive movement is statistically more likely to be followed by another positive movement or vice versa. On the other hand, the anti-persistent process implies that a positive movement is more statistically probable to be followed by a negative movement and vice versa [35].

3. Methods

In this section, we present three methods of detection of the long-range dependence in time series. Classical rescaled range analysis is presented as the oldest and currently again very used method. As the alternatives robust to the presence of the short-range dependence in series, we present modified rescaled range and rescaled variance analyses.

3.1. Classical rescaled range analysis

Rescaled range analysis (R/S) is the oldest Hurst exponent estimation method and was proposed by Harold E. Hurst while working as an engineer in Egypt [18] and further adjusted by [28]. In the procedure and for financial time series, one divides the series of T continuous returns into N adjacent sub-periods of length υ so that $N\upsilon = T$. For each sub-period I_n, a rescaled range of a profile X_{t,I_n} (cumulative deviations from an arithmetic mean) is calculated as R_{I_n}/S_{I_n}, where $R_{I_n} = max_{I_n}(X_{t,I_n}) - min_{I_n}(X_{t,I_n})$ is a range of the corresponding profile and S_{I_n} is a standard deviation of the corresponding returns. The same procedure is applied to each sub-period of the given length and an average rescaled range is calculated [30]. The rescaled ranges scale as

$$(R/S)_\upsilon \approx c\upsilon^H$$

with varying υ where c is a constant [33]. Linear relationship in a double-logarithmic scale indicates a power scaling [37]. To uncover the scaling law, an ordinary least squares regression on logarithms of each side of the equation is applied and H is estimated.

V statistic, which is used for a cycles detection, a stability testing of Hurst exponent or a change in scaling behavior (crossover) detection, is defined as

$$V_\upsilon = \frac{(R/S)_\upsilon}{\sqrt{\upsilon}}$$

and converges to a distribution defined as $F_V(x) = 1 + 2\sum_{k=1}^{\infty}(1 - 4k^2x^2)e^{-2(kx)^2}$ for independent processes [26, 18, 30]. The distribution has a mean $\mathbb{E}(F_V) = \sqrt{\pi/2}$ and a variance $var(F_V) = \pi(\pi - 3)/6$ with critical values of 0.809 and 1.862 for 5% level of significance. Therefore, if the value of V statistic lies outside of the interval (0.809, 1.862), the null hypothesis of no long-range dependence is rejected. The statistic is constant, increasing and decreasing with increasing scale υ for no long-range dependence, persistence and anti-persistence, respectively.

3.2. Modified rescaled range analysis

As R/S analysis presented above (usually called classical) is biased by a presence of short-term memory, [26] proposed modified rescaled range analysis ($M - R/S$) which differs from the classical one by a use of a modified standard deviation (S^M). S^M is defined with a use of auto-covariances of the original series γ_j in the selected sub-interval up to lag ξ as

$$S_{I_n}^M = \sqrt{S_{I_n}^2 + 2\sum_{j=1}^{\xi}\gamma_j\left(1 - \frac{j}{\xi+1}\right)}. \tag{1}$$

Thus, R/S turns into a special case of $M - R/S$ with $\xi = 0$. The distribution of the modified V statistic then converges to F_V not only for independent processes but also for short-range dependent ones. Choice of the correct lag ξ is critical for the estimation of the modified rescaled ranges [36, 34]. [26] suggested optimal lag based on the first-order autocorrelation coefficient of the original series $\rho(1)$ defined as (where [] is the nearest lower integer operator)

$$\xi^* = \left[\left(\frac{3\upsilon}{2}\right)^{\frac{1}{3}}\left(\frac{2\rho(1)}{1-\rho(1)^2}\right)^{\frac{2}{3}}\right]. \tag{2}$$

One then gets the estimates of the modified rescaled ranges for the set scale $\upsilon = T$, constructs the V statistics and compares it with critical values of above shown distribution F_V with the null hypothesis of no long-range dependence.

3.3. Rescaled variance analysis

Rescaled variance analysis (V/S) is quite novel method proposed by [16] as a modified version of KPSS statistic [22], which is usually used for testing of stationarity but was shown to have good power for long-range dependent series [23, 24]. The procedure is very similar to modified rescaled range analysis and differs in a use of a sample variance of the profile of the series instead of the range. As an alternative to the V statistic, the M statistic is defined as

$$M_\upsilon = \frac{var(X_{t,I_n})}{\upsilon(S_{I_n}^M)^2}.$$

Note that the modified standard deviation (Equation 1) is used so that the method is robust to short-range dependence as well. The M statistic of independent or short-range dependent processes converges to a distribution defined as $F_M(x) = 1+2\sum_{k=1}^{\infty}(-1)^k e^{-2k^2\pi^2 x}$. The distribution has a mean $\mathbb{E}(F_M) = 1/12$ and a variance $var(F_M) = 1/360$ with critical values of 0.0249 and 0.222 for 5% level of significance. Similarly to both rescaled range analyses, if the value of M statistic lies outside of the interval (0.0249, 0.222), the null hypothesis of no long-range dependence is rejected.

Variance of the M statistic is much lower than the one of R/S and $M - R/S$ so that the confidence intervals are much narrower.

The use of the sample variance instead of the range (in the case of R/S and $M-R/S$) or the partial sum of squares (in the case of $KPSS$) has better statistical power against long-range dependence even in squares and thus volatility in the financial sense [16]. [4] showed that V/S can be also used as an estimator of Hurst exponent by the means of the scaling form

$$M_{\upsilon} \approx c\upsilon^{2H}.$$

Similarly to R/S and $M - R/S$, one needs to follow the steps of the R/S if estimating H or set $\upsilon = T$ if only testing the presence of long-range dependence in the process.

4. Research methodology

4.1. Moving block bootstrap

The bootstrap method [13] was develop to deal with the statistical properties of small samples. The basic notion behind the procedure is resampling with replacement from the original series and repeated estimation of a specific parameter or statistic. By shuffling, a distribution of the original series remains unchanged while possible dependencies are distorted [9]. Hypothesis can be then tested with use of confidence intervals based on the bootstrapped estimates. For our purposes, simple bootstrap is not enough as the shuffling rids us not only from the long-range dependence but the short-range one as well. [32] proposed a modified method which retains the short-term dependence characteristics but lacks the long one - the moving block bootstrap with pre-whitening and post-blackening.

In the procedure, the time series $\{x_t\}_{t=1}^T$ is firstly pre-whitened by a specific process - usually $AR(p)$ - and residuals ε_t are obtained. These are further centered around the average value $\bar{\varepsilon}$ so that the centered residuals $\{\varepsilon_t^c\}_{t=1}^T$ are defined as $\varepsilon_t^c = \varepsilon_t - \bar{\varepsilon}$. The series $\{\varepsilon_t^c\}_{t=1}^T$ is then divided into m blocks of length ζ while $m\zeta = T$. The blocks are then reshuffled and post-blackened with the use of the model from the pre-whitening part and residuals ε_t^c to form new bootstrapped time series $\{x_t^b\}_{t=1}^T$. Such time series retains the short-range dependence, potential heteroskedasticity and trends as well as the distribution of the original time series. However, for small enough ζ, the long-range dependencies are torn. The examined statistic is then estimated on the new time series. The procedure is repeated B times so that the confidence intervals for hypothesis testing of no long range dependence can be constructed. The original estimate of the statistic of the original time series is then compared with the bootstrapped confidence intervals.

4.2. Parameters choice

As it is a presence of the long-range dependence in process rather than its specific form or level which is important for financial implications, we estimate V and M statistics derived from R/S, $M - R/S$ and V/S rather than Hurst exponent H.

Such procedure is used for several reasons. Firstly, there are many different methods for Hurst exponent estimation and there is no consensus which one is the best - for comparison, see [37, 8, 17, 30, 1, 6]. Secondly, Hurst exponent H based on different methods is only an estimate of the real Hurst exponent and has quite wide confidence intervals for finite samples [37, 21, 8]. And thirdly, Hurst exponent estimation usually assumes the underlying process to be either fractional Brownian motion (fBm) or from a family of autoregressive fractionally integrated moving averages ($ARFIMA$) or from a family of fractionally integrated (generalized) autoregressive conditional heteroskedasticity models ($FI(G)ARCH$). However, the complex dynamics of the underlying process can be even more complicated.

For the moving block bootstrap, we use $AR(1)$ process for pre-whitening and post-blackening, which is standard in finance literature [14, 19], and $\zeta = 10$. The lengths of the time series T are then taken as a multiple of 10. Such choice of ζ should be sufficient for ridding of the potential long-range dependence while the other properties remain similar to the original process. Moreover, the length of 10 observations is usually suggested as a minimum interval in various long-range dependence detection techniques [29]. For each time series, we construct 2.5% and 97.5% confidence intervals from corresponding quantiles of 1000 bootstrapped time series ($B = 1000$). The procedure is repeated for lags $\xi = 0, 1, \ldots, 10$ while $\xi \in \mathbb{Z}_0^+$.

5. Data

We apply classical and modified rescaled range and rescaled variance analyses on returns and volatility of BUX, PX and WIG. Let $P_{i,t}$ be a closing value of the index i, for $i = 1, 2, 3$ with respect to three examined indices, at time t for $t = 0, \ldots, T_i$, where T_i is time series length of index i. Continuous returns of the index i at time t are then defined as $r_{i,t} = log(P_{i,t}/P_{i,t-1})$ for $t = 1, \ldots, T_i$. As a measure of volatility, we use the absolute returns defined as $|r_{i,t}|$ for index i at time t.

The examined period is the longest for PX starting from 7.7.1997, followed by BUX starting from 26.7.2001 and the shortest sample is the one of WIG from 14.10.2003. The last examined trading day for all three indices is 30.6.2009. Evolution of the index values and corresponding returns are presented in Figure 1. Basic descriptive statistics are summed in Table 1. From the table, we can see that the returns are in hand with basic risk/return notion of the financial theory - PX is the least risky but with the lowest average return whereas WIG offered the highest average return with the highest risk measured by standard deviation. Moreover, all returns are negatively skewed and leptokurtic and thus not normally distributed according to Jarque-Bera statistic as well [20]. The result is confirmed by QQ-plots, which are shown together with histograms in Figure 2. Basic descriptive statistics are in hand with the stylized facts of the financial markets [7]. Note that WIG seems to be quite close to normality when compared to the other two indices. Further, stationarity cannot be rejected by KPSS test [22] for any examined index.

The results of the basic statistical analysis uncover the importance of bootstrap method for the construction of the confidence intervals opposed to the ones based on standard normal distribution such as in [37]. Methods can be sensitive to different distributions (see e.g. [33]) and the bootstrapping avoids such pitfall.

6. Results and discussion

Table 2 and Figure 3 sum estimated V statistics for both R/S (with $\xi = 0$) and $M-R/S$ for the whole examined period together with the confidence intervals based on the moving block bootstrap. Table 2 also shows estimates for the optimal lag ξ^* based on Equation 2 in bold font. For the returns, the optimal lag was proposed as the 4th, the 5th and the 2nd for BUX, PX and WIG, respectively. Similarly for the volatility, the optimal lag was estimated as the 10th, the 12th and the 4th for BUX, PX and WIG, respectively. Such propositions indicate that there is the strongest short-range dependence in PX and quite weak one in WIG.

The estimates for all examined lags ξ based on both methods show very homogeneous results - on one hand, there is no long-range dependence in returns of BUX, PX and WIG and on the other hand, the volatility of all examined indices is long-range dependent. Such outcomes put some previously cited results into question since the indices of the Central Europe are usually marked as the less developed and less efficient markets. For such markets, the persistence was usually claimed to be found. However, majority of the studies do not take statistical properties of the tested series into consideration and only compare the estimates of Hurst exponent H or V statistic with the critical values. Therefore, the results should be taken with caution or even reexamined with respect to statistical properties of the series as well as possible short-range dependence.

Last but not least for the rescaled range analyses, Figure 3 also compares the confidence intervals based on the bootstrapping with the ones based on [26], which are correct given the fact that the lag ξ was chosen correctly. The bootstrapped confidence intervals based on the time series of the returns are approximately equal to the ones of [26] for all examined lags. As this is true even for $\xi = 0$, the short-range dependence is either not present at all or very weak and does not bias the estimates of classical R/S. The situation is more interesting for volatility. The bootstrapped confidence intervals do not collide enough up till between lags 8 and 9. For lower lags ξ, both bootstrapped critical values are much higher than the ones of [26]. This implies that the optimal lag is not the one of Equation 2 but the one where the confidence intervals based on the bootstrapping and the ones based on [26] equal. Nevertheless, the difference between the two gives us some additional information. For BUX and PX, the volatility is long-range dependent as well as short-range dependent with $AR(1)$ being the correct choice. WIG, on the other hand, showed the lowest level of short-range dependence, which is reflected in the lowest departure from the confidence intervals of [26], but it still has the highest estimates of V statistics for all lags ξ compared to the other two. Such result implies two possible interpretations - either the short-range dependence should be modeled by some other model or the long-range dependence is stronger in the volatility of WIG than it is in the volatility of BUX and PX. The latter option seems to be more rational as the difference between the two types of confidence intervals is the lowest for WIG as it was mentioned earlier.

Similarly, Table 3 and Figure 4 show the results for V/S method. The notation holds. The results are very similar and support the fact that there is no long-range dependence found in the series of the returns whereas the volatility is long-range dependent. However, there are several important differences. The results for the returns of PX are not as clear as for the case of rescaled range analyses. For lags 0, 2, 4 and 5, the comparison with bootstrapped confidence intervals implies long-range dependent series. The results are weakened by the fact that the estimates of M statistics are between the critical values of the asymptotic F_M distribution, except for lag 0. Further, the bootstrapped confidence intervals seem to vary more with changing lag ξ when compared to the ones of modified rescaled range analysis. Such result contradicts the claim of [16] that the estimates of M statistics are less dependent on the choice of ξ than the estimates of V statistics.

For the long-range dependence in the volatility of the stock indices, the results are the same. WIG shows the strongest persistence with low influence of the short-range dependence. For BUX and PX, both the short and the long-range dependence are quite strong as the estimates of M statistics are far from the upper critical value but decrease steadily.

7. Conclusion

We researched on the presence of the long-range dependence in the returns and the volatility of BUX, PX and WIG between years 1997 and 2009. From several techniques for the long-range dependence detection, we chose classical and modified rescaled range analyses together with rescaled variance analysis and tested the long-range dependence on the basis of V and M statistics. To avoid the potential short-range dependence, distributional, heteroskedasticity and trend complications, we applied the moving block bootstrap with pre-whitening and post-blackening for the construction of the confidence intervals for the hypothesis testing. On one hand, we showed that there was no significant long-range dependence in returns of all examined indices. On the other hand, significant long-range dependence was detected in the volatility of all three indices. For BUX and PX, both significant short and long-range dependence were detected whereas for WIG, the short-range dependence was only weak but the long-range dependence was the strongest from the three indices.

We have also discussed on a possibility of finding the optimal lag for modified rescaled range procedure. Moreover, the results for the returns are contradictory with several studies which claim that developing markets are persistent. However, majority of these studies either do not use the confidence intervals at all or only the ones based on standard normal distribution. Therefore, the results of such studies should be taken with caution or even reexamined and reinterpreted.

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Table 1: Descriptive statistics of BUX, PX and WIG returns

	BUX	PX	WIG
Mean	0,000438	0,0002	0,00016
Min	-0,12649	-0,16185	-0,08443
Max	0,13178	0,12364	0,08155
Standard deviation	0,01682	0,01535	0,01688
Skewness	-0,22025	-0,48709	-0,2773
Excess kurtosis	7,14249	12,27884	2,7021
Jarque-Bera statistic	4160,749	18863,73	465,702
- p-value	0,0000	0,0000	0,0000
KPSS	0.3996	0.2240	0.4677
Observations	1950	2980	1460
Start date	26.7.2001	7.7.1997	14.10.2003
End date	30.6.2009	30.6.2009	30.6.2009

Table 2: V statistics and bootstrapped confidence intervals

		returns			volatility		
	ξ	V	$P_{97.5}$	$P_{0.25}$	V	$P_{97.5}$	$P_{0.25}$
BUX	0	1.7978	1.9076	0.8578	5.1971*	3.3785	1.5066
	1	1.7319	1.8909	0.8052	4.5694*	2.9306	1.3111
	2	1.7488	1.8683	0.8172	4.1498*	2.6644	1.1927
	3	1.7696	1.9243	0.8399	3.8433*	2.4675	1.0984
	4	**1.7556**	**1.9229**	**0.8457**	3.6085*	2.3184	1.0622
	5	1.7341	1.8685	0.8496	3.4136*	2.3168	1.0174
	6	1.7231	1.8355	0.8375	3.2356*	2.1442	0.9917
	7	1.7314	1.8547	0.8477	3.0838*	2.1131	0.9540
	8	1.7327	1.9073	0.8309	2.9545*	2.0287	0.9056
	9	1.7245	1.8843	0.8505	2.8462*	1.9742	0.9108
	10	1.7175	1.8588	0.8442	**2.7514***	**2.0329**	**0.8636**
PX	0	1.8083	1.8146	0.8292	5.3791*	3.5733	1.5431
	1	1.7313	1.7676	0.7809	4.6937*	3.0088	1.2965
	2	1.7274	1.8226	0.8070	4.2041*	2.7325	1.2158
	3	1.7344	1.8025	0.7861	3.8634*	2.5371	1.1483
	4	1.7273	1.7825	0.8025	3.6027*	2.4011	1.0632
	5	**1.7164**	**1.8606**	**0.7957**	3.3873*	2.3298	0.9863
	6	1.7099	1.7943	0.7817	3.2108*	2.2459	0.9581
	7	1.7105	1.8159	0.7980	3.0608*	2.1473	0.9295
	8	1.7085	1.7869	0.8021	2.9330*	2.0862	0.8686
	9	1.7082	1.8393	0.8195	2.8223*	1.9996	0.9090
	10	1.7029	1.8544	0.7954	2.7237*	2.0228	0.8651
WIG	0	1.7275	1.8327	0.8266	5.5596*	2.9259	1.2842
	1	1.6942	1.7381	0.8059	5.2617*	2.8235	1.1756
	2	**1.7008**	**1.7774**	**0.7801**	4.9856*	2.5225	1.1301
	3	1.6937	1.7514	0.7500	4.6606*	2.4410	1.0539
	4	1.6894	1.7597	0.8057	**4.3816***	**2.3968**	**1.0051**
	5	1.6900	1.7238	0.7913	4.1307*	2.2174	0.9584
	6	1.6892	1.7578	0.7737	3.9156*	2.1753	0.9465
	7	1.6971	1.7453	0.7660	3.7368*	2.1025	0.9068
	8	1.7042	1.7611	0.7955	3.5852*	2.0843	0.9252
	9	1.7043	1.7373	0.7570	3.4458*	2.0804	0.8959
	10	1.7037	1.7659	0.8005	3.3211*	1.9788	0.8713

Table 3: M statistics and bootstrapped confidence intervals

		returns			volatility		
	ξ	V	$P_{97.5}$	$P_{0.25}$	V	$P_{97.5}$	$P_{0.25}$
BUX	0	0.1937	0.2302	0.0288	2.6271*	0.8499	0.1027
	1	0.1798	0.2207	0.0241	2.0309*	0.6843	0.0774
	2	0.1833	0.2282	0.0273	1.6750*	0.5334	0.0644
	3	0.1877	0.2442	0.0268	1.4367*	0.4764	0.0583
	4	0.1847	0.2472	0.0265	1.2665*	0.4325	0.0518
	5	0.1802	0.2503	0.0291*	1.1334	0.3904	0.0486
	6	0.1780	0.2342	0.0271	1.0183*	0.3765	0.0461
	7	0.1797	0.2379	0.0276	0.9250*	0.3299	0.0408
	8	0.1800	0.2371	0.0268	0.8491*	0.2985	0.0397
	9	0.1783	0.2177	0.0273	0.7880*	0.2994	0.0383
	10	0.1768	0.2365	0.0282	0.7363*	0.2763	0.0327
PX	0	0.2406*	0.2175	0.0260	2.9085*	0.8822	0.1028
	1	0.2205	0.2423	0.0229	2.2144*	0.6964	0.0718
	2	0.2195*	0.2083	0.0234	1.7765*	0.5745	0.0606
	3	0.2213	0.2254	0.0253	1.5002*	0.4781	0.0530
	4	0.2195*	0.2157	0.0218	1.3046*	0.3908	0.0451
	5	0.2167*	0.2140	0.0240*	1.1534	0.3861	0.0411
	6	0.2151	0.2182	0.0223	1.0362*	0.3499	0.0398
	7	0.2152	0.2198	0.0228	0.9417*	0.3369	0.0356
	8	0.2148	0.2085	0.0227	0.8647*	0.3047	0.0355
	9	0.2147	0.2165	0.0238	0.8007*	0.2857	0.0351
	10	0.2133	0.2166	0.0244	0.7457*	0.2910	0.0342
WIG	0	0.1949	0.2032	0.0248	3.2730*	0.6333	0.0701
	1	0.1875	0.2000	0.0235	2.9317*	0.5315	0.0574
	2	0.1890	0.1937	0.0220	2.6322*	0.4984	0.0565
	3	0.1874	0.2076	0.0242	2.3002*	0.4424	0.0459
	4	0.1864	0.1877	0.0238	2.0330*	0.3960	0.0461
	5	0.1866	0.1849	0.0236*	1.8068	0.3498	0.0419
	6	0.1864	0.2202	0.0214	1.6236*	0.3318	0.0357
	7	0.1881	0.1916	0.0233	1.4787*	0.3160	0.0351
	8	0.1897	0.2166	0.0229	1.3611*	0.2946	0.0324
	9	0.1898	0.2016	0.0242	1.2573*	0.3039	0.0338
	10	0.1896	0.2064	0.0249	1.1680*	0.2618	0.0325

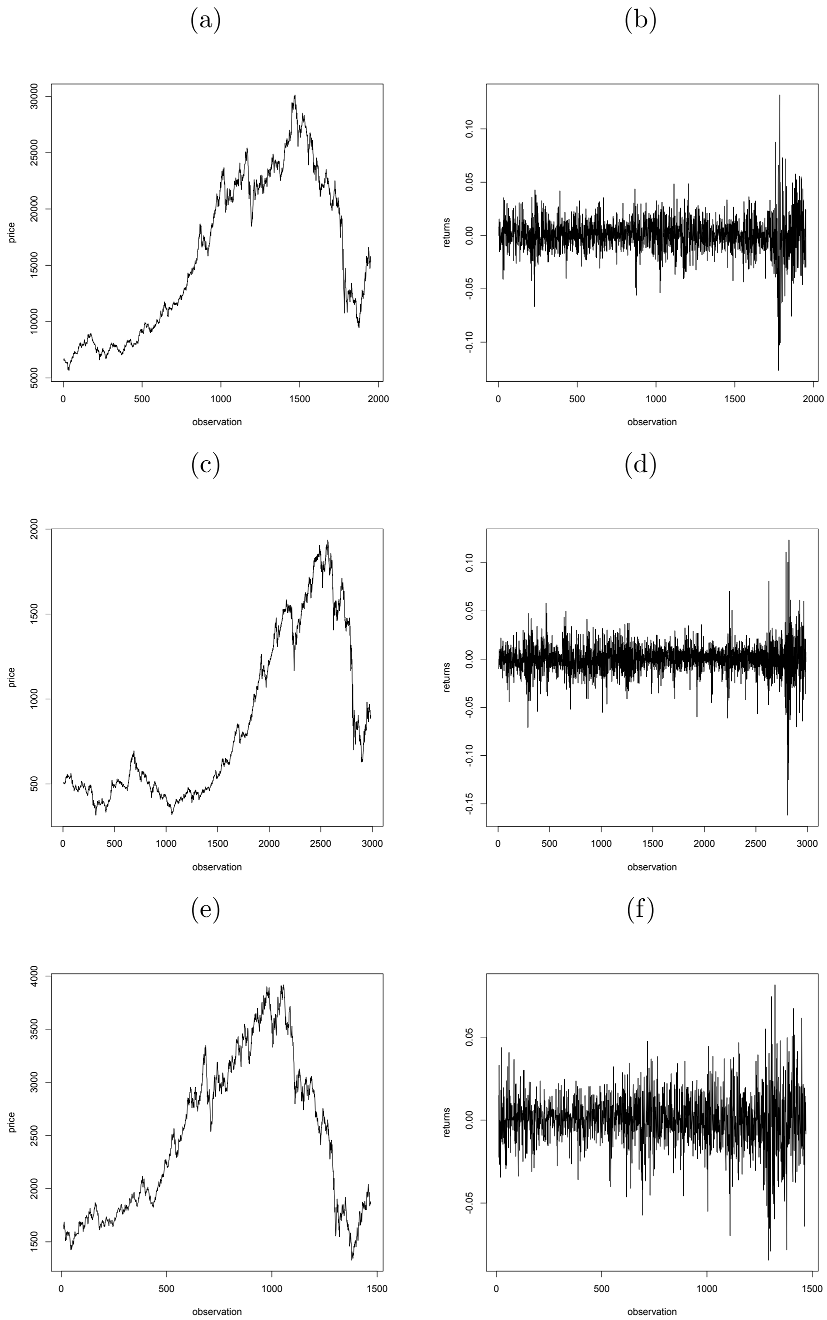

Figure 1: Evolution of index values and returns for BUX, PX and WIG. BUX, PX and WIG are shown on the first, second and third row, respectively, with prices in the first column and returns in the second one. BUX is shown for the period 26.7.2001 - 30.6.2009, PX for the period 7.7.1997 - 30.6.2009 and WIG for the period 14.10.2003 - 30.6.2009.

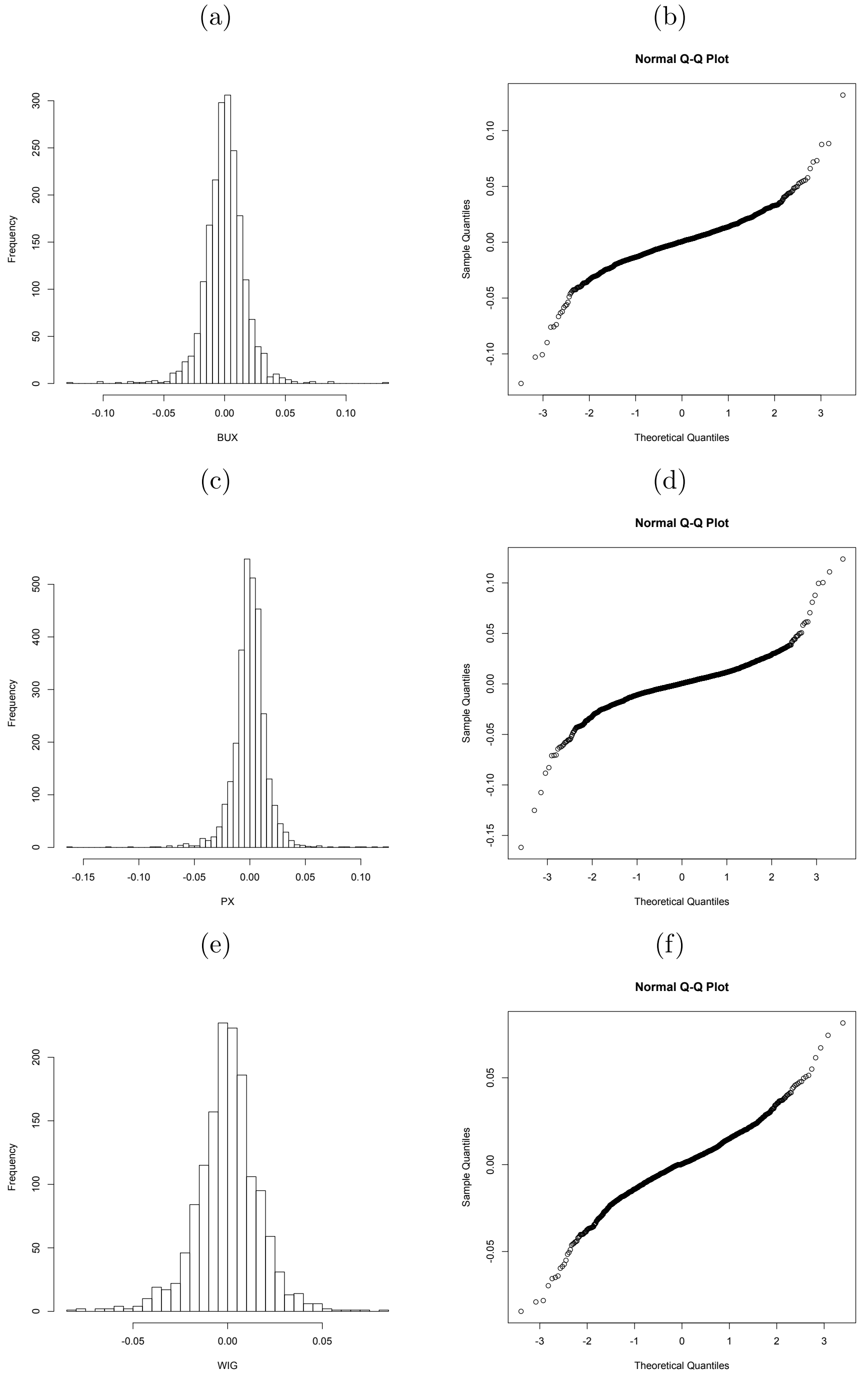

Figure 2: Histograms and QQ-plots for BUX, PX and WIG returns. BUX, PX and WIG are shown on the first, second and third row, respectively, with histograms in the first column and QQ-plots in the second one. QQ-plots compare the quantiles of respective returns with the quantiles of the normal distribution.

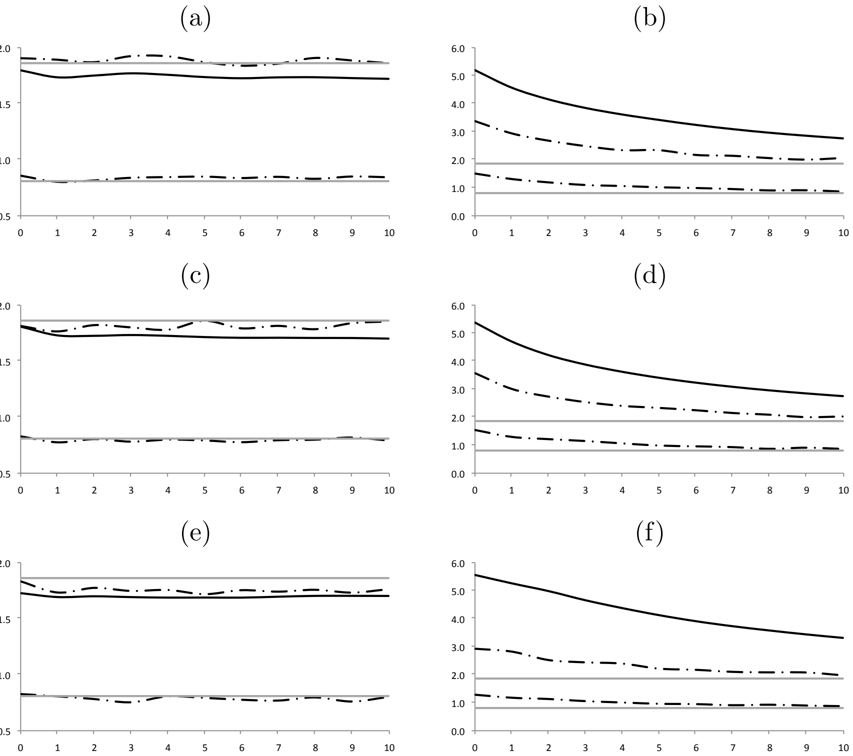

Figure 3: Estimates of V statistics for returns (first column) and volatility (second column) for BUX (first row), PX (second row) and WIG (third row). The x-axis represents the lag ξ used. Solid black curve represents the actual estimates of V statistics, dashed black curves show the 2.5% and 97.5% confidence intervals based on moving block bootstrap procedure and grey solid lines represent the 2.5% and 97.5% confidence intervals of [26].

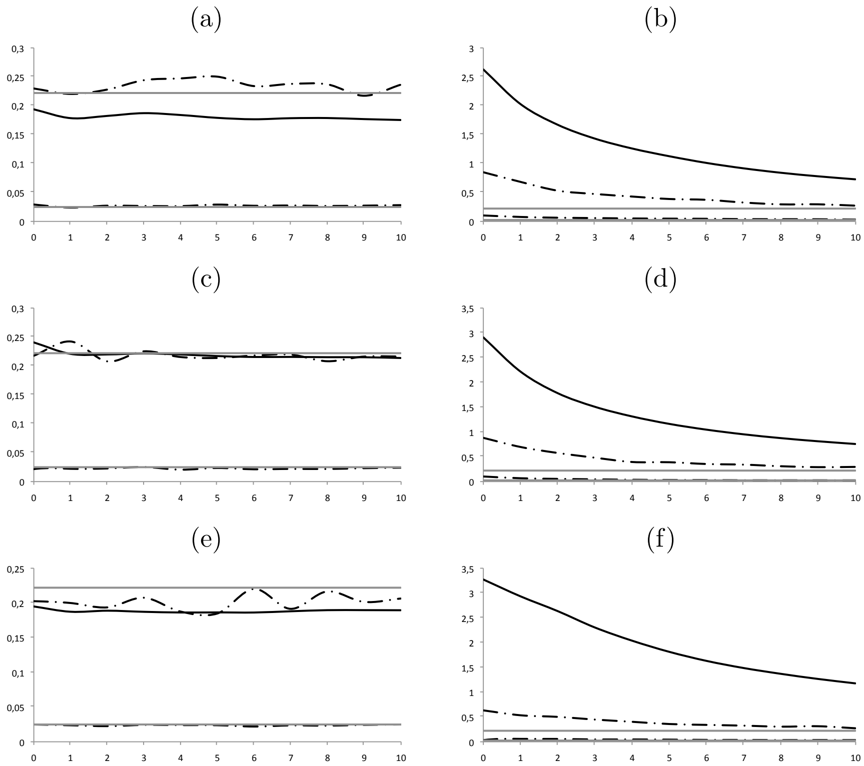

Figure 4: Estimates of M statistics for returns (first column) and volatility (second column) for BUX (first row), PX (second row) and WIG (third row). The x-axis represents the lag ξ used. Solid black curve represents the actual estimates of M statistics, dashed black curves show the 2.5% and 97.5% confidence intervals based on moving block bootstrap procedure and grey solid lines represent the 2.5% and 97.5% confidence intervals of [16].